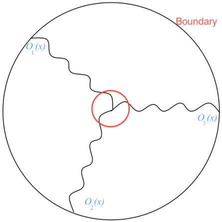
Semiclassical 3-point functions for short operators



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Maths of String and Gauge Theory
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Introduction

- $\mathcal{N} = 4$ SYM "solved" in the planar limit, $\lambda = g_{YM}^2 N$
  - Spectrum of gauge invariant operators determined by asymptotic Bethe ansatz Staudacher (2004), BS(2005), *op. cit.*
  - Complications (winding effects, finite size). These are handled by TBA, Y-system, Hirota, FiNLIE AF (2008), GKV (2009), AF(2009) *op. cit.*

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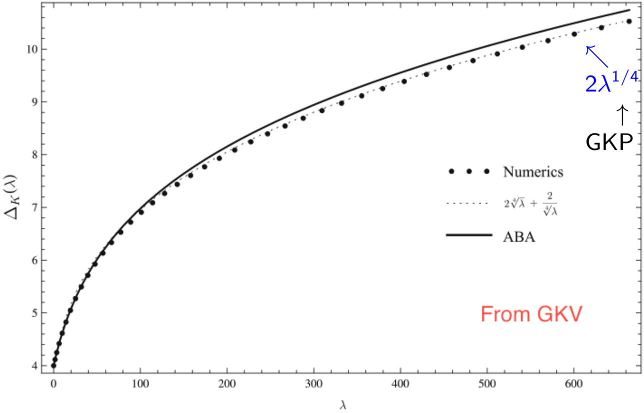
    - Complications (winding effects, finite size). These are handled by TBA, Y-system, Hirota, FiNLIE AF (2008), GKV (2009), AF(2009) *op. cit.*
- Konishi operator: $\mathcal{O} = \mathrm{tr}(\phi^I\phi^I) \sim \mathrm{tr}[Z,W]^2$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta(\lambda)}}$$



From GKV

$$\underset{\substack{\uparrow\\ \text{GKP}}}{2\lambda^{1/4}} + \underset{\substack{\uparrow\\ \text{1-loop w-s}}}{\frac{2}{\lambda^{1/4}}} + \dots$$

Introduction (cont)

- $\mathcal{N} = 4$ SYM is a CFT
- Primary operator $\mathcal{O}(x)$: $K^\mu \mathcal{O}(0) = 0$.
- Besides the spectrum we have three-point functions for primary ops. Large N limit:

$$\mathcal{Z}_1^{-1/2}\mathcal{Z}_2^{-1/2}\mathcal{Z}_3^{-1/2}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{1}{N}\frac{\mathcal{C}_{123}}{|x_{12}|^{2\alpha_3}|x_{23}|^{2\alpha_1}|x_{31}|^{2\alpha_2}}$$

$$\alpha_1 = \tfrac{1}{2}(\Delta_2+\Delta_3-\Delta_1) \qquad \alpha_2 = \tfrac{1}{2}(\Delta_3+\Delta_1-\Delta_2) \qquad \alpha_3 = \tfrac{1}{2}(\Delta_1+\Delta_2-\Delta_3)$$

- C_{123} is protected for 3 chiral primaries ($Q\mathcal{O}(0) = 0$) LMRS (1998)
- What about nonchiral?

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- Semiclassical string calculation for large λ:
    - "Two heavy, one light" Zarembo (2010), CMSZ (2010) ...
    - Three heavy Janik, Wereszczynski (2011), Buchbinder Tseytlin (2011)

"Heavy" are long classical strings

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  "Heavy" are long classical strings
- Konishi is neither classical nor light. It is a short heavy string state.
  Can one find the 3-point function ($\lambda \gg 1$) for three Konishi operators?

Introduction (cont)

- General idea: Use the flat-space limit.
- Konishi is a "short" operator: dual to a short string state in AdS/CFT
- Short string: doesn't see the curvature of $AdS_5 \times S^5$ $(R = 1)$
  GKP (1998) String size $\sim \sqrt{\alpha'} = \frac{1}{\lambda^{1/4}} \ll 1$

$$m^2 = \Delta^2 - d\Delta \approx \Delta^2$$

Flat space: $$m^2 = \frac{4n}{\alpha'} = 4n\,\lambda^{1/2}$$

AdS/CFT dictionary: $$m^2 = \Delta^2 - d\Delta \approx \Delta^2$$

$$\Delta \approx 2\sqrt{n}\,\lambda^{1/4}$$

$n = 1$ for Konishi

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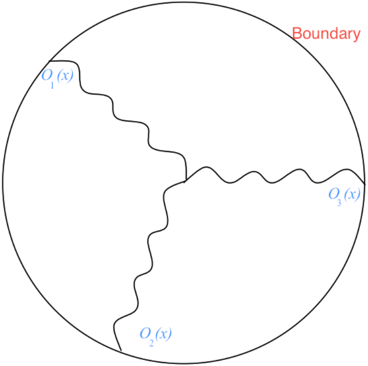
- Can the flat space idea be applied to three-point functions?

Still the Introduction – Witten diagrams



FMMR: Integrate over the intersection point.

Use sugra coupling $\mathcal{G}_{123}$

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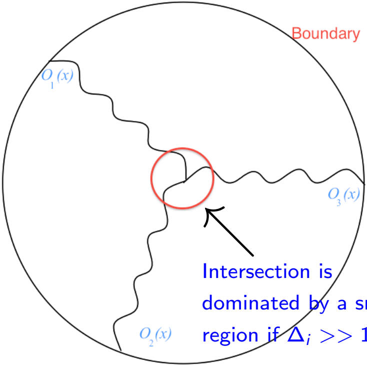

Intersection is dominated by a small region if $\Delta_i >> 1$

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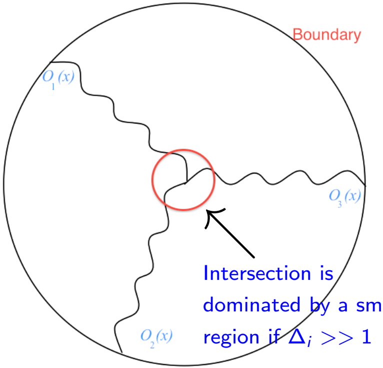

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- For Konishi ops we need to find the couplings
- Treat the states as particles as they come in from the boundary, strings when they enter the intersection region

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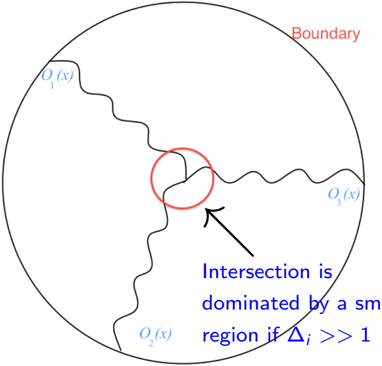

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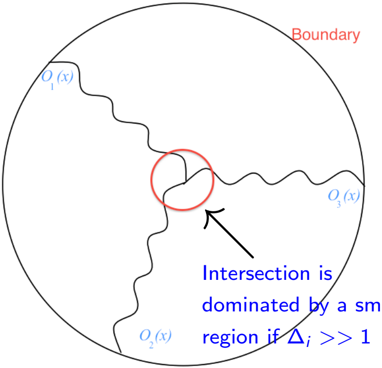


Intersection is dominated by a small region if $\Delta_i >> 1$

- For Konishi ops we need to find the couplings
- Treat the states as particles as they come in from the boundary, strings when they enter the intersection region
- Small interaction region: use flat space string vertex operators.
- But
  - What are the momenta?
  - Which vertex operator?

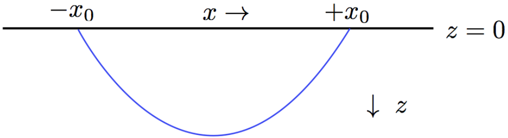
Outline

- Introduction ✓
- Computing 2-point functions Janik, Surowka, Wereszczynski (2010)
- 3-point functions semi-classically
- String vertex operators (Including “Konishi-like” states)
- Concluding remarks

2-point functions

Path integral for a point particle (boundary to boundary) (JSW (2010))

$$\langle \mathcal{O}_\Delta(x_0)\mathcal{O}_\Delta(-x_0)\rangle \; =$$



Euclidean action in AdS_{d+1}:

$$S = \frac{1}{2}\int_{-1}^{+1} ds \left[\frac{\dot{x}^\mu \dot{x}_\mu + \dot{z}^2}{z^2} e^{-1} + \Delta^2 e\right]$$

n.b. We are using Δ^2 and not m^2 in S.
Reparm. inv.: Fix $e(s) = E$. $x(\pm 1) = \pm x_0$, $z(\pm 1) = \epsilon$.

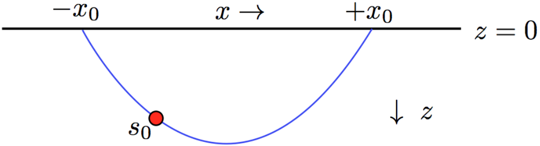
$$\mathcal{Z} = \int \prod_{s=-1}^{s=1} \frac{\mu\, \mathcal{D}x^\mu(s)\mathcal{D}z(s)\mathcal{D}\, e(s)}{V_{diff}}\, e^{-S} = \int dE \prod_{s=-1}^{s=1} \mu\, \mathcal{D}x^\mu(s)\mathcal{D}z(s)(Jac)\, e^{-S}$$

Minimizing S:

$$\begin{aligned} x(s) &= R\tanh(\kappa s) \\ z(s) &= R\,\mathrm{sech}(\kappa s) \\ x_\perp^\mu &= 0 \end{aligned} \quad \boxed{\begin{array}{l} \lim \epsilon \rightarrow 0 \\ R \approx x_0 \\ \kappa \approx \log\left(\frac{2x_0}{\epsilon}\right) \end{array}} \quad \begin{array}{l} S_{\mathrm{cl}} = \kappa^2 E^{-1} + \Delta^2 E \\ E = \kappa/\Delta,\; S_{cl} = 2\kappa\Delta \end{array} \Rightarrow \boxed{\mathcal{Z} \sim \left|\frac{2x_0}{\epsilon}\right|^{-2\Delta}}$$

2-point functions

Break up the path integral into two parts: $x^\mu(s_0) = x^\mu$, $z(s_0) = z$.



$$\int \left(dE_- \prod_{s<s_0} \mu\, \mathcal{D}x^\mu(s)\mathcal{D}z(s)(Jac) \right) \frac{z^{-d-1} d^d x\, dz}{V_{Gauge}} \left(dE_+ \prod_{s>s_0} \mu\, \mathcal{D}x^\mu(s)\mathcal{D}z(s)(Jac) \right) e^{-S},$$

$$S_{cl} = -\Delta \log \left(\frac{z\,\epsilon}{z^2 + (x - x_0)^2 + x_\perp^2} \right) - \Delta \log \left(\frac{z\,\epsilon}{z^2 + (x + x_0)^2 + x_\perp^2} \right)$$

Minimize wrt x^μ and z gives a point on the previous trajectory

Fluctuations over the joining point give:

transverse: $\left(\frac{\pi}{\Delta}\right)^{\frac{d-1}{2}}$

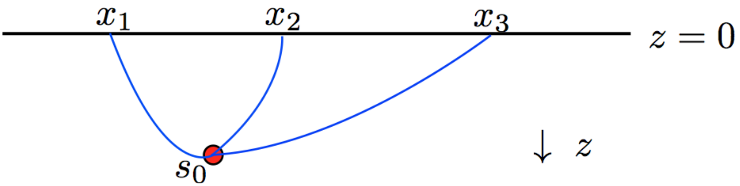
longitudinal: $\left(\frac{\pi}{\Delta}\right)^{\frac{1}{2}} \times$ zero mode (Zero mode is $\sim V_{gauge}$)

$$\boxed{\mathcal{Z} \approx \mathcal{C}_- \frac{2\pi^{d/2}}{\Delta^{d/2-1}} \mathcal{C}_+ \left| \frac{2x_0}{\epsilon} \right|^{-2\Delta}}$$

3-point functions

Three incoming particles meeting at the joining point: $x^\mu(s_0) = x^\mu$, $z(s_0) = z$.

$\mathcal{Z}_{123} \equiv$



$$S_{\rm cl}(x^\mu, z) = -\sum_{i=1}^{3} \Delta_i \log\left(\frac{z\,\epsilon}{z^2 + (x - x_i)^2}\right)$$

$$\Pi_{\mu\,i} = -i\,E_i^{-1}\frac{\dot{x}_\mu}{z^2} \qquad \Pi_{z\,i} = -i\,E_i^{-1}\frac{\dot{z}}{z^2}$$

Minimize wrt x^μ and z leads to conservation of momentum:

$$\sum_{i=1}^{3} \Pi_{\mu,i} = 0 \qquad \sum_{z,i} \Pi_{z,i} = 0$$

$$\Pi_i \cdot \Pi_i \equiv G^{mn}\Pi_{m\,i}\Pi_{n\,i} = -\Delta_i^2\,.$$

$$\Pi_1{\cdot}\Pi_2 = \frac{1}{2}\left(\Delta_1^2 + \Delta_2^2 - \Delta_3^2\right), \quad \Pi_2{\cdot}\Pi_3 = \frac{1}{2}\left(\Delta_2^2 + \Delta_3^2 - \Delta_1^2\right), \quad \Pi_3{\cdot}\Pi_1 = \frac{1}{2}\left(\Delta_3^2 + \Delta_1^2 - \Delta_2^2\right)$$

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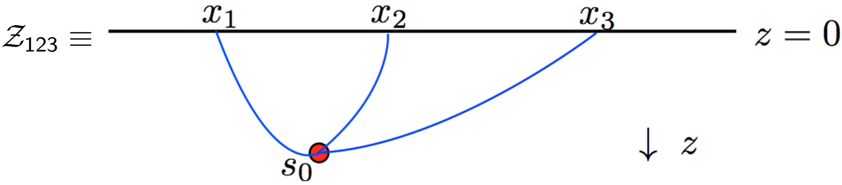


$$\boxed{S_{\text{cl}} = \log\left(|x_{12}|^{2\alpha_3}|x_{23}|^{2\alpha_1}|x_{31}|^{2\alpha_2}\right) - \sum_i(\alpha_i \log\alpha_i - \Delta_i \log\Delta_i) - \Sigma\log\Sigma}$$

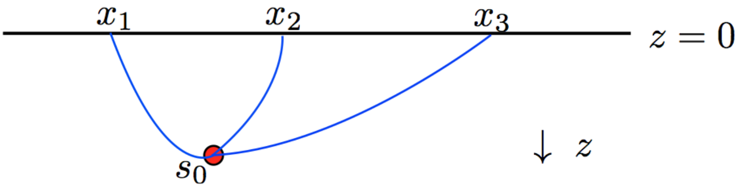
where

$$\alpha_1 = \tfrac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1)\,, \qquad \alpha_2 = \tfrac{1}{2}(\Delta_3 + \Delta_1 - \Delta_2)\,, \quad \alpha_3 = \tfrac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)\,,$$
$$\Sigma = \alpha_1 + \alpha_2 + \alpha_3\,,$$

(See also Klose & McLoughlin (2011))

3-point functions

$\mathcal{Z}_{123} \equiv$



$$Z_{123} = \boxed{\frac{\pi^{\frac{d+1}{2}}}{\sqrt{2}} \left(\frac{\Delta_1\Delta_2\Delta_3}{\alpha_1\alpha_2\alpha_3\Sigma^{d+1}} \right)^{1/2} \mathcal{C}_{1-}\mathcal{C}_{2-}\mathcal{C}_{3-}} \boxed{\frac{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}\Sigma^{\Sigma}}{\Delta_1^{\Delta_1}\Delta_2^{\Delta_2}\Delta_3^{\Delta_3}} \frac{1}{|x_{12}|^{2\alpha_3}|x_{23}|^{2\alpha_1}|x_{31}|^{2\alpha_2}}} \mathcal{G}_{123}$$

Fluctuations:

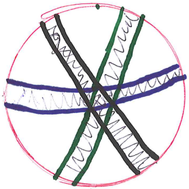
$e^{-S_{cl}}$

$$\begin{aligned} \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3) \rangle &= \frac{1}{\mathcal{C}_{1-}\mathcal{C}_{2-}\mathcal{C}_{3-}} \frac{(\Delta_1\Delta_2\Delta_3)^{\frac{d-2}{4}}}{2\sqrt{2}\pi^{\frac{3d}{4}}} \mathcal{Z}_{123} \\ &= \frac{C_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}|x_{31}|^{\Delta_3+\Delta_1-\Delta_2}} \end{aligned}$$

$$C_{123} \approx \frac{\pi^{\frac{2-d}{4}}}{4} \frac{(\Delta_1\Delta_2\Delta_3)^{d/4}}{(\alpha_1\alpha_2\alpha_3\Sigma^{d+1})^{1/2}} \frac{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}\Sigma^{\Sigma}}{\Delta_1^{\Delta_1}\Delta_2^{\Delta_2}\Delta_3^{\Delta_3}} \mathcal{G}_{123}$$

Intersection region: $\Delta r_{AdS} \sim \left(\frac{\Delta_1\Delta_2\Delta_3}{\alpha_1\alpha_2\alpha_3\Sigma^{d+1}} \right)^{1/2(d+1)} \sim \frac{1}{\Delta^{1/2}} \sim \frac{1}{\lambda^{1/8}} \ll 1$

S^5 contribution (see also Buchbinder and Tseytlin (2011))



$\longleftarrow$(Also intersects at antipode.)

Reduce to S^2 subspace, $\sum_{I=1}^{3} X_I^2 = 1$. Choose three normalized wave-functions

$$
\begin{aligned}
\psi_{J_1}(\vec{X}) &= \frac{\sqrt{(J_1+1)(J_1+2)}}{\sqrt{2\pi^3}}(X_1+iX_2)^{J_1} \\
\psi_{J_2}(\vec{X}) &= \frac{\sqrt{(J_2+1)(J_2+2)}}{\sqrt{2\pi^3}}(X_1-i(\cos\chi X_2+\sin\chi X_3))^{J_2} \\
\psi_{J_3}(\vec{X}) &= \frac{\sqrt{(J_3+1)(J_3+2)}}{\sqrt{2\pi^3}}(X_1-i(\cos\chi' X_2-\sin\chi' X_3))^{J_3}
\end{aligned}
$$

Dominant where $\vec{J}_1+\vec{J}_2+\vec{J}_3=0$ $\qquad \langle\psi_{J_1}\psi_{J_2}\psi_{J_3}\rangle \approx \frac{1}{2\pi^2}\frac{(J_1J_2J_3)^{3/2}}{(\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3\tilde{\Sigma}^5)^{1/2}}$

$\Delta r_{S^5} \sim \left(\frac{J_1J_2J_3}{\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3\tilde{\Sigma}^5}\right)^{1/10} \sim \frac{1}{J^{1/2}}$ $\Rightarrow$ Intersection region small if $J_i \gg 1$

String vertex operators: Strategy

- Let $\Delta_i \gg J_i \gg 1$ (Konishi has $J_i = 0$)
- Particles are wave-packets with wavelength $\sim \frac{1}{\lambda^{1/4}}$, spread $\sim \frac{1}{\lambda^{1/8}}$
- $\Rightarrow$ Treat as plane-waves in the intersection region
- Particle momentum: $k_{Mi} = (\Pi_{\mu i}, \Pi_{zi}; \vec{J}_i)$, $M = 0 \dots 9$
- Usual factors of $(2\pi)^{10}\delta^{10}(k_1 + k_2 + k_3)$ are replaced with overlaps
- Use level one flat string vertex operators ($k^2 = -4/\alpha' = -4\sqrt{\lambda}$) ($\vec{J}_i$ can be set to zero for this part.)

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- Which vertex operator do we use?
- At level one there are 128×128 physical states in the NS-NS sector and 128×128 physical states in the R-R sector!!!

String vertex operators: Flattening the conformal algebra

Which vertex operators to use:

Conformal algebra in d dimensions:

$$[D, P_\mu] = -iP_\mu \qquad [D, M_{\mu\nu}] = 0 \qquad [D, K_\mu] = +iK_\mu$$
$$[M_{\mu\nu}, P_\lambda] = -i(\eta_{\mu\lambda} P_\nu - \eta_{\lambda\nu} P_\mu) \qquad [M_{\mu\nu}, K_\lambda] = -i(\eta_{\mu\lambda} K_\nu - \eta_{\lambda\nu} K_\mu)$$
$$[P_\mu, K_\nu] = 2i(M_{\mu\nu} - \eta_{\mu\nu} D)\,.$$

Rewrite: $[P_\mu \pm K_\mu, P_\nu \pm K_\nu] = \pm 4iM_{\mu\nu}$, $[P_\mu + K_\mu, P_\nu - K_\nu] = 4i\eta_{\mu\nu} D$
$[D, P_\mu + K_\mu] = -i(P_\mu - K_\mu)$, $[D, P_\mu - K_\mu] = -i(P_\mu + K_\mu)$

Rescale: $\tilde{P}_\mu = \epsilon(P_\mu + K_\mu)/2$, $\tilde{P}_d = \epsilon D$, $M_{d\mu} = (P_\mu - K_\mu)/2$ $\qquad \lim \epsilon \to 0$

$\Rightarrow$ Poincaré algebra in $d + 1$ dimensions:

$[\tilde{P}_\mu, \tilde{P}_\nu] = 0$, $[M_{\mu\nu}, \tilde{P}_\lambda] = -i(\eta_{\mu\lambda} \tilde{P}_\nu - \eta_{\lambda\nu} \tilde{P}_\mu)$ $\qquad \mu = 0 \ldots d$

String vertex ops: Flattening the superconformal algebra

$\mathcal{N} = 4$ Superconformal algebra in 4 dimensions:

$$
\begin{aligned}
\{Q_{\alpha\, a}, \widetilde{Q}^b_{\dot\alpha}\} &= \gamma^\mu_{\alpha\dot\alpha}\delta_a{}^b P_\mu\,, \qquad \{S^a_\alpha, \widetilde{S}_{\dot\alpha\, b}\} = \gamma^\mu_{\alpha\dot\alpha}\delta^a{}_b K_\mu \qquad a, b = 1\ldots 4, \quad \alpha, \dot\alpha = 1, 2\\
\{Q_{\alpha\, a}, S^b_\beta\} &= -i\varepsilon_{\alpha\beta}\sigma^{IJ}{}_a{}^b R_{IJ} + \gamma^{\mu\nu}_{\alpha\beta}\delta_a{}^b M_{\mu\nu} - \frac{1}{2}\varepsilon_{\alpha\beta}\delta_a{}^b\, D \qquad I, J = 1\ldots 6\\
\{\widetilde{Q}^a_{\dot\alpha}, \widetilde{S}_{\dot\beta\, b}\} &= +i\varepsilon_{\dot\alpha\dot\beta}\sigma^{IJ\, a}{}_b\, R_{IJ} + \gamma^{\mu\nu}_{\dot\alpha\dot\beta}\delta^a{}_b M_{\mu\nu} - \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\delta^a{}_b D\\
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\end{aligned}
$$

Rescale: $\hat{Q}^A_{L,R} = \sqrt{\frac{\epsilon}{2}}\big(Q_{\alpha a} \pm i S^b_\alpha \Gamma^6_{ba}, \widetilde{Q}^a_{\dot\alpha} \mp i\widetilde{S}_{\dot\alpha\, b}\Gamma^{6ab}\big), \qquad \tilde{P}_J = \epsilon R_{J6}$

⇒ 10d $\mathcal{N} = 2$ SuperPoincaré algebra (after including $M_{\mu I}$)

$$
\{\hat{Q}^A_{L,R}, \hat{Q}^B_{L,R}\} = \Gamma^{AB}_M \tilde{P}^M \qquad \{\hat{Q}^A_L, \hat{Q}^B_R\} = 0 \qquad M = 0\ldots 9
$$

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\{Q_{\alpha\, a}, \widetilde{Q}^{b}_{\dot\alpha}\} &= \gamma^{\mu}_{\alpha\dot\alpha}\delta_a{}^b P_\mu\,, \qquad \{S^a_\alpha, \widetilde{S}_{\dot\alpha\, b}\} = \gamma^{\mu}_{\alpha\dot\alpha}\delta^a{}_b K_\mu \qquad a, b = 1\ldots 4, \quad \alpha, \dot\alpha = 1, 2 \\
\{Q_{\alpha\, a}, S^b_\beta\} &= -i\varepsilon_{\alpha\beta}\sigma^{IJ}{}_a{}^b R_{IJ} + \gamma^{\mu\nu}_{\alpha\beta}\delta_a{}^b M_{\mu\nu} - \frac{1}{2}\varepsilon_{\alpha\beta}\delta_a{}^b\, D \qquad I, J = 1\ldots 6 \\
\{\widetilde{Q}^a_{\dot\alpha}, \widetilde{S}_{\dot\beta\, b}\} &= +i\varepsilon_{\dot\alpha\dot\beta}\sigma^{IJ\,a}{}_b\, R_{IJ} + \gamma^{\mu\nu}_{\dot\alpha\dot\beta}\delta^a{}_b M_{\mu\nu} - \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\delta^a{}_b D \\
\{Q_{\alpha\, a}, \widetilde{S}_{\dot\beta\, b}\} &= \{\widetilde{Q}^a_{\dot\alpha}, S^b_\beta\} = \{Q_{\alpha\, a}, Q_{\alpha\, b}\} = \{\widetilde{Q}^a_{\dot\alpha}, \widetilde{Q}^b_{\dot\alpha}\} = \{S^a_\alpha, S^b_\alpha\} = \{\widetilde{S}_{\dot\alpha\, a}, \widetilde{S}_{\dot\alpha\, b}\} = 0\,.
\end{aligned}$$

Rescale: $\hat{Q}^A_{L,R} = \sqrt{\tfrac{\epsilon}{2}}\big(Q_{\alpha a} \pm iS^b_\alpha \Gamma^6_{ba}, \widetilde{Q}^a_{\dot\alpha} \mp i\widetilde{S}_{\dot\alpha\, b}\Gamma^{6ab}\big), \qquad \tilde{P}_J = \epsilon R_{J6}$

$\Rightarrow$ 10d $\mathcal{N} = 2$ SuperPoincaré algebra (after including $M_{\mu I}$)

$$\{\hat{Q}^A_{L,R}, \hat{Q}^B_{L,R}\} = \Gamma^{AB}_M \tilde{P}^M \qquad \{\hat{Q}^A_L, \hat{Q}^B_R\} = 0 \qquad M = 0\ldots 9$$

Primary Operator: $\quad S^b_\alpha \mathcal{O}(0) = \widetilde{S}_{\dot\alpha\, b}\mathcal{O}(0) = 0$

Flat Space: $\quad \hat{Q}_L = \hat{Q}_R$

String vertex ops: $Q_L = Q_R$

Mixing of NS-NS and R-R modes:

$$Q_L(|NS\rangle \otimes |NS\rangle + |R\rangle \otimes |R\rangle) = |R\rangle \otimes |NS\rangle + |NS\rangle \otimes R\rangle$$

$$Q_R(|NS\rangle \otimes |NS\rangle + |R\rangle \otimes |R\rangle) = |NS\rangle \otimes |R\rangle + |R\rangle \otimes NS\rangle$$

$Q_L = Q_R$ requires a mixture of both sets of fields

String vertex ops: Level one

Relevant vert. ops. can be found in the ancient literature (FMS (1985); Kostelecky et. al (1987); Koh, Troost, van Proeyen (1987))

NS sector (Left movers, $(-1$ picture)): $\alpha_{LMN}\psi^L\psi^M\psi^N$ 84 states

$\sigma_{MN} i\partial X^M \psi^N$ 44 states

R sector (Left movers, $(-\frac{1}{2}$ picture)): $(V^A_M i\partial X^M \Theta_A + \rho_{MA}\psi^M \not{\psi}\Theta^A)$ 128 states

$$\rho^M = -\tfrac{1}{8} V^M \not{k} + \tfrac{1}{36}(V\cdot k)\Gamma^M$$

$$k^L \alpha_{LMN} = k^M \sigma_{MN} = \sigma^M{}_M = k^M V^A_M = \not{V}_M = 0$$

$Q_L = Q_R$ requires all three combinations. Scalar vertex ops:

$$\begin{aligned}
V_1^{(-1,-1)}(z,\bar{z}) &= g_s\left(\tfrac{2}{\alpha'}\right)\sigma_{MN;\tilde{M}\tilde{N}}\,\psi^M(z)\partial X^N \tilde{\psi}^{\tilde{M}}(\bar{z})\bar{\partial}X^{\tilde{N}}\,e^{ik\cdot X}e^{-\phi-\tilde{\phi}}\,,\\
V_2^{(-1,-1)}(z,\bar{z}) &= g_s\,\alpha_{MNL;\tilde{M}\tilde{N}\tilde{L}}\,\psi^M(z)\psi^N(z)\psi^L(z)\,\psi^{\tilde{M}}(\bar{z})\psi^{\tilde{N}}(\bar{z})\psi^{\tilde{L}}(\bar{z})\,e^{ik\cdot X}e^{-\phi-\tilde{\phi}}\,.\\
\sigma_{MN;\tilde{M}\tilde{N}} &= \left(\tfrac{1}{2}(\hat{\eta}_{M\tilde{M}}\hat{\eta}_{N\tilde{N}}+\hat{\eta}_{M\tilde{N}}\hat{\eta}_{N\tilde{M}}) - \tfrac{1}{9}\,\hat{\eta}_{MN}\hat{\eta}_{\tilde{M}\tilde{N}}\right)\\
\alpha_{MNL;\tilde{M}\tilde{N}\tilde{L}} &= \tfrac{1}{3!}(\hat{\eta}_{M\tilde{M}}\hat{\eta}_{N\tilde{N}}\hat{\eta}_{L\tilde{L}} - \text{ perms})
\end{aligned}$$

where $\hat{\eta}_{MN} \equiv \eta_{MN} - \frac{k_M k_N}{k^2}$.

$$\begin{aligned}
V_3^{(-1/2,-1/2)}(z,\bar{z}) &= g_s\left(\tfrac{2}{\alpha'}\right)^{1/2} e^{ikX}e^{-\phi/2-\tilde{\phi}/2}\\
&\quad\times(i\,\bar{\partial}X^M\tilde{\Theta}^A - \left(\frac{\alpha'}{16}\right)\tilde{\psi}^M\not{k}\tilde{\not{\psi}}\tilde{\Theta})C\not{k}(\hat{\eta}_{MN} - \tfrac{1}{9}\hat{\Gamma}_M\hat{\Gamma}_N)(i\,\partial X^N\Theta^B - \left(\frac{\alpha'}{16}\right)\psi^N\not{k}\not{\psi}\Theta)
\end{aligned}$$

where $\hat{\Gamma}^M = \Gamma^M - \not{k}k^M/k^2$.

String vertex ops: Level one

One combination satisfies $Q_L = Q_R$:

$$V = \kappa\,(V_1 + \ V_2 - \ V_3)$$

$$\kappa = (44 + 84 + 128)^{-1/2} = 1/16$$

String vertex ops: Level one

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$\langle V_2^{(-1,-1)} V_3^{(-1/2,-1/2)} V_3^{(-1/2,-1/2)} \rangle$
etc.

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etc.

Nasty combinatorics: Work in progress

Discussion

- The three-point functions for "short" operators can be effectively approximated using flat space vertex operators.
- We have shown the general method how one can find the 3-point for three Konishi-like operators (hope to know the result soon).
- Since the vertex functions do not depend strongly on $\vec{J}$ if $J \ll \Delta$, the only dependence on J in the 3-point functions is in the wave-function overlaps. *It is tempting to replace the overlaps with the overlap of the strictly $J = 0$ wave-functions* (those functions that are constant on S^5), rendering a result for the true Konishi operator.

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- The three-point functions for “short” operators can be effectively approximated using flat space vertex operators.
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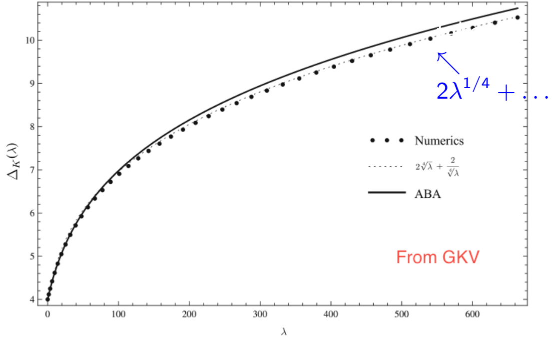
- Since the vertex functions do not depend strongly on $\vec{J}$ if $J \ll \Delta$, the only dependence on J in the 3-point functions is in the wave-function overlaps. *It is tempting to replace the overlaps with the overlap of the strictly $J = 0$ wave-functions* (those functions that are constant on S^5), rendering a result for the true Konishi operator.



$2\lambda^{1/4} + \ldots$

From GKV

Thank you!!

String vertex ops: Massless example

Massless vertex ops. $\Rightarrow$ Chiral Primary ops:

$$k^M = (\vec{\Delta}; \vec{J}) \quad \Rightarrow \quad k^2 = 0$$

NS-NS vertices: $V^{-1,-1}(z,\bar{z}) = -g_s \varepsilon_{MN} \psi^M \tilde{\psi}^N e^{-\phi-\tilde{\phi}} e^{ik\cdot X}$, $\quad k^M \varepsilon_{MN} = 0$

$$V^{0,0}(z,\bar{z}) = -g_s \varepsilon_{MN} \tfrac{2}{\alpha'} \big(i\partial X^M + \tfrac{\alpha'}{2} k\cdot\psi\psi^M\big)\big(i\bar{\partial} X^M + \tfrac{\alpha'}{2} k\cdot\tilde{\psi}\tilde{\psi}^M\big) e^{ik\cdot X},$$

R-R vertices: $V^{-1/2,-1/2}(z,\bar{z}) = g_s \left(\frac{\alpha'}{2}\right)^{1/2} t^{AB} \Theta_A \tilde{\Theta}_B e^{-\frac{1}{2}\phi - \frac{1}{2}\tilde{\phi}} e^{ik\cdot X}$, $\quad t \not{k} = 0$

$Q_{L,R}$ made from zero-mom R vertices: $Q^{(-1/2)}_{R,A} = \big(\tfrac{2}{\alpha'}\big)^{1/4} \oint \tfrac{d\bar{z}}{2\pi i} \tilde{\Theta}_A e^{-\frac{1}{2}\tilde{\phi}}$

$$Q^{(+1/2)}_{L,A} = \big(\tfrac{2}{\alpha'}\big)^{3/4} \oint \tfrac{dz}{2\pi i} i\partial X_M \Gamma^M_{AB} \Theta^B \, e^{+\frac{1}{2}\phi}$$

$$Q^{(+1/2)}_{L,A} V^{-1,-1}(z,\bar{z}) = \tfrac{1}{\sqrt{2}} g_s \varepsilon_{MN} \left(\frac{\alpha'}{2}\right)^{1/4} (\not{k}\Gamma^M)^B{}_A \theta_B \tilde{\psi}^N e^{-\frac{1}{2}\phi - \tilde{\phi}} e^{ikX}$$

$$Q^{(-1/2)}_{R,A} V^{-1/2,-1/2}(z,\bar{z}) = \tfrac{1}{\sqrt{2}} g_s t^{BC} \left(\frac{\alpha'}{2}\right)^{1/4} (\mathcal{C}\Gamma_N)_{CA} \Theta_B \tilde{\psi}^N e^{-\frac{1}{2}\phi - \tilde{\phi}} e^{ikX}$$

$$\Rightarrow \qquad \varepsilon_{MN} (\not{k}\Gamma^M)^B{}_A = t^{BC} (\mathcal{C}\Gamma_N)_{CA}$$

Only solution: $\quad \varepsilon_{MN} = \eta_{MN} - \frac{k_M k_N}{k^2}, \qquad t^{AB} = (\not{k}\mathcal{C})^{AB}$

n.b. This choice does not exactly satisfy $Q_L = Q_R$, but it is satisfied up to spurious terms (pure gauge)